

Testing

The Science of Quarks and Gluons

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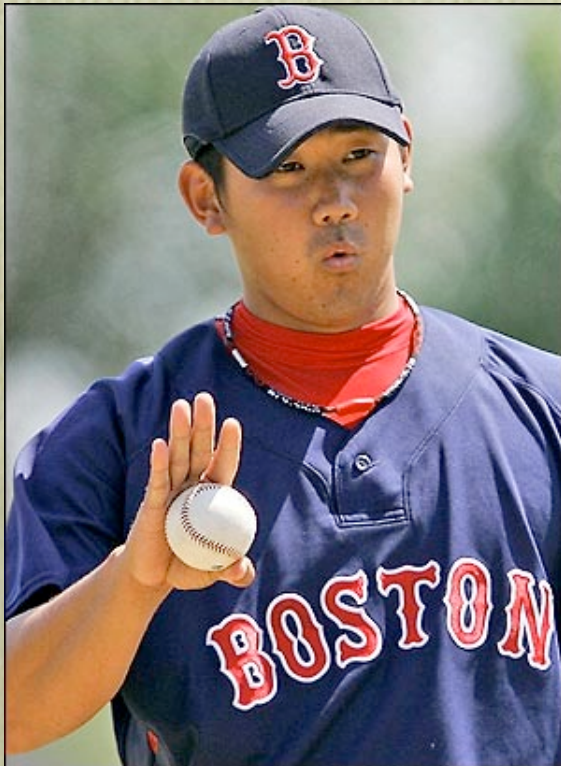
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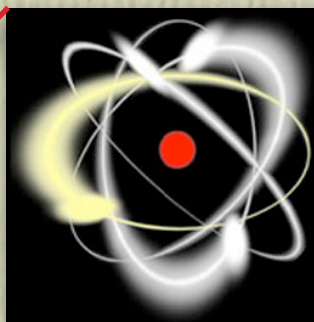
The Science of Us!



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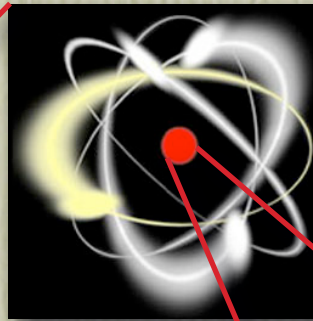
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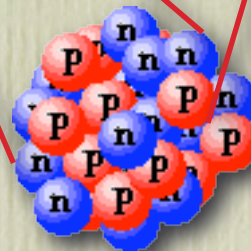
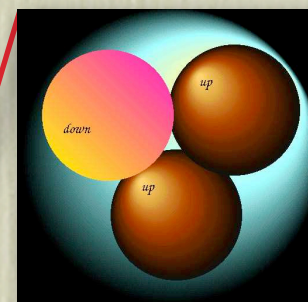
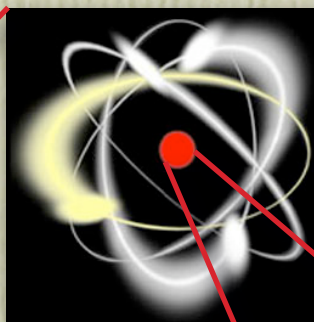
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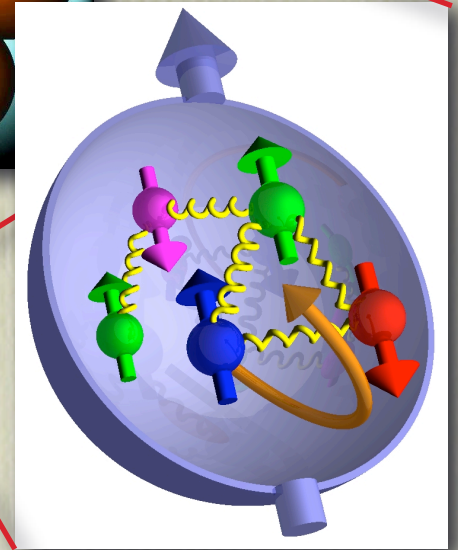
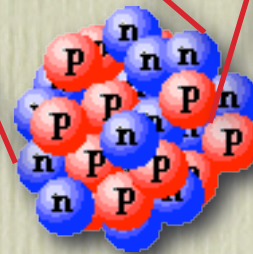
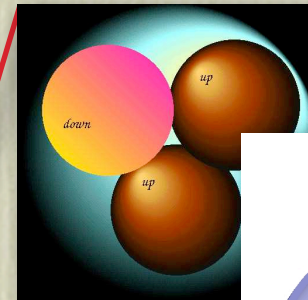
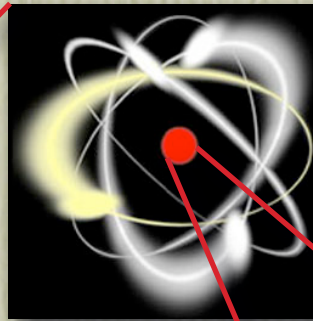
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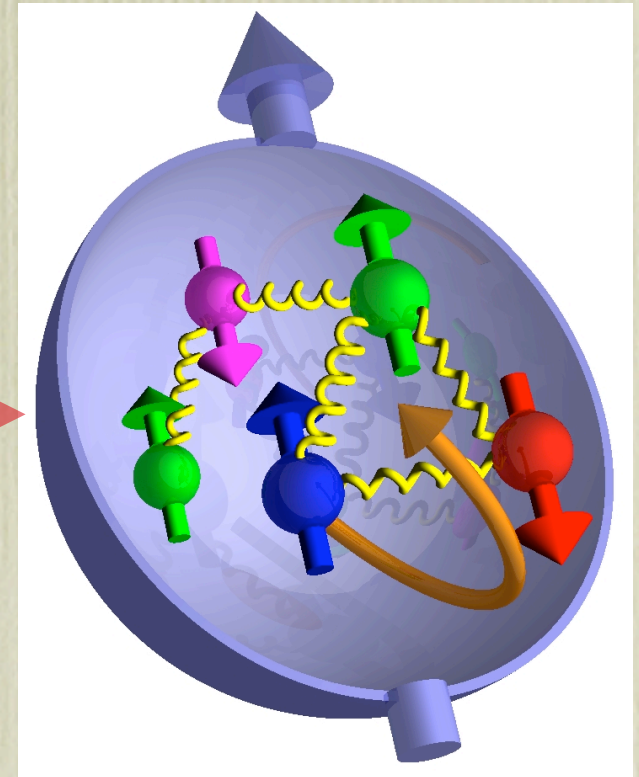
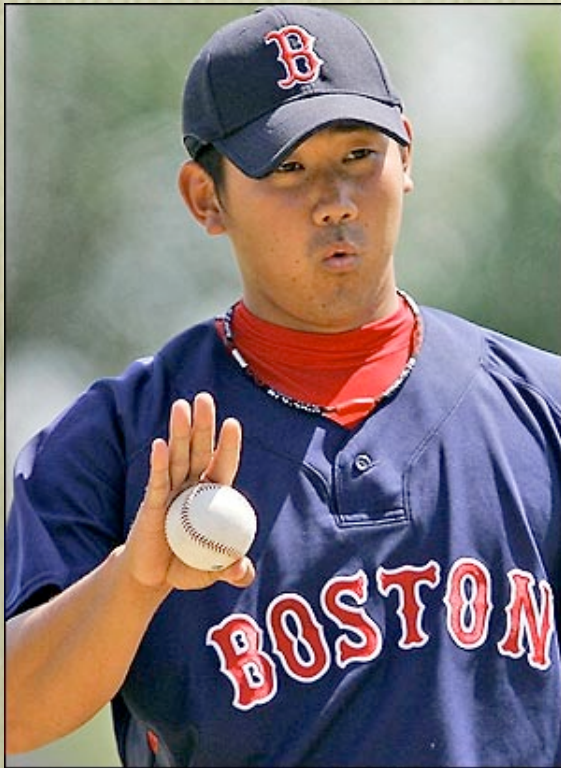
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QCD

Quantum Chromo Dynamics

$$\mathcal{L} = -\frac{1}{4} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \bar{q}(iD_{\mu}\gamma^{\mu} + m)q$$

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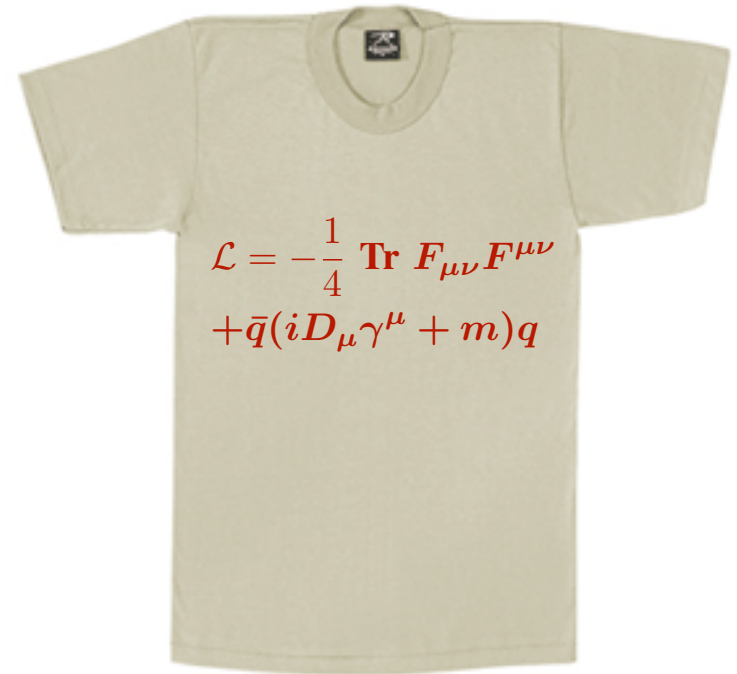
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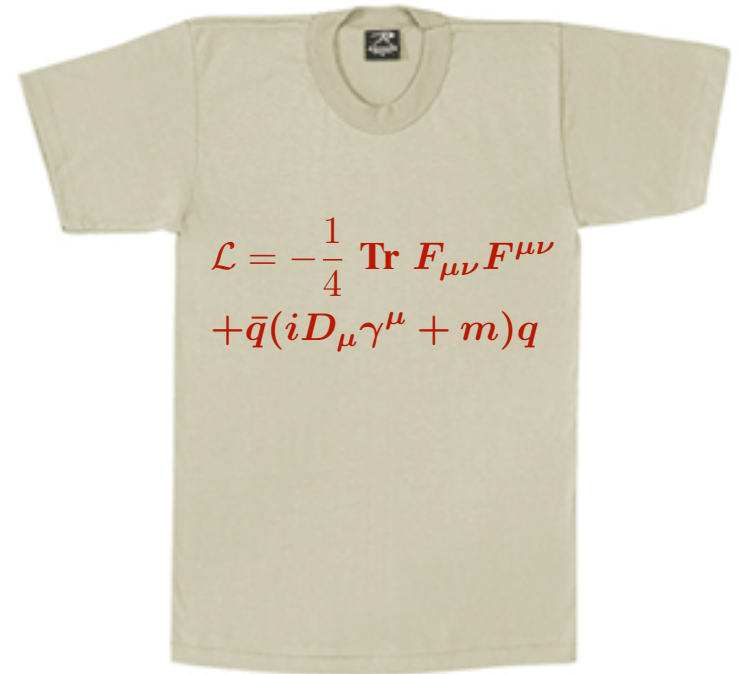
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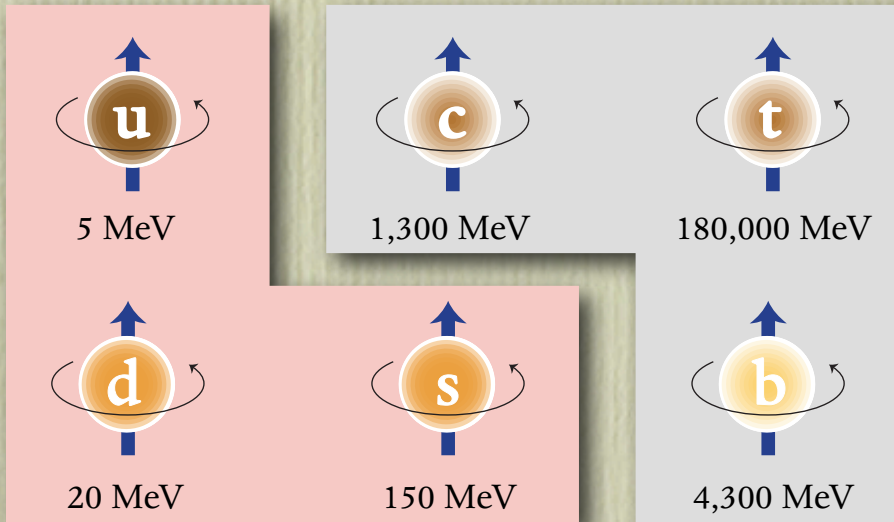


QCD is the only complete, correct, and internally consistent description of any aspect of Nature that we have managed to construct!

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Quarks



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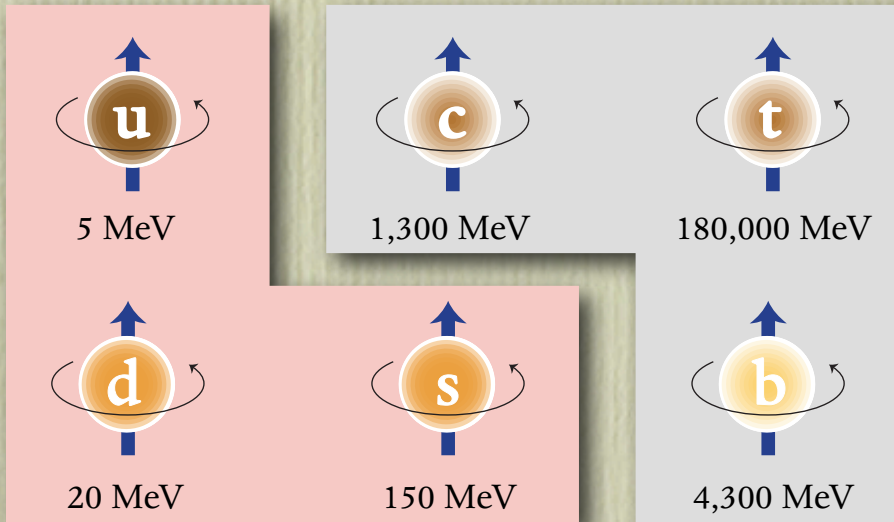
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QCD \approx 150 to 250 MeV

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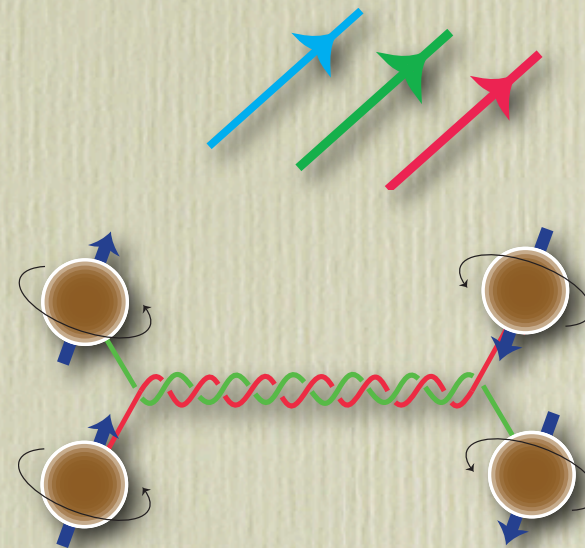
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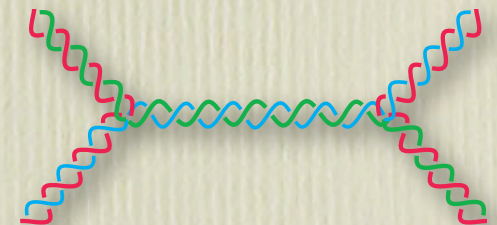
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Color

Quarks carry three color “charges” that couple to eight long range forces carried by “gluons”.



Quark-quark force



Gluon-gluon force

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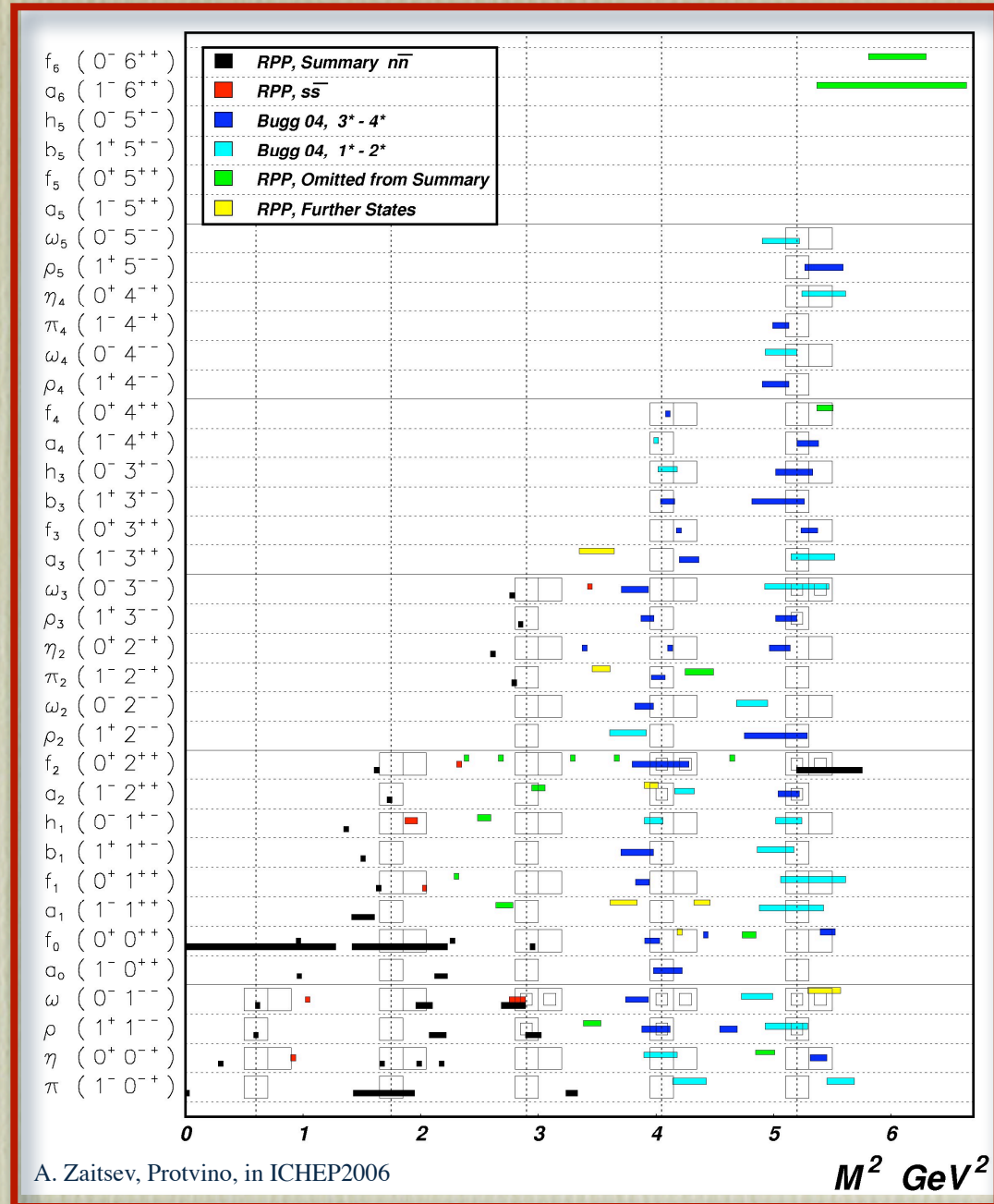
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- Rich phenomenology amenable to experiment!

ONE EXAMPLE: UNEXPLAINED REGULARITIES IN THE SPECTRUM OF LIGHT, NON-STRANGE MESONS.



Emergent phenomena in a precision environment

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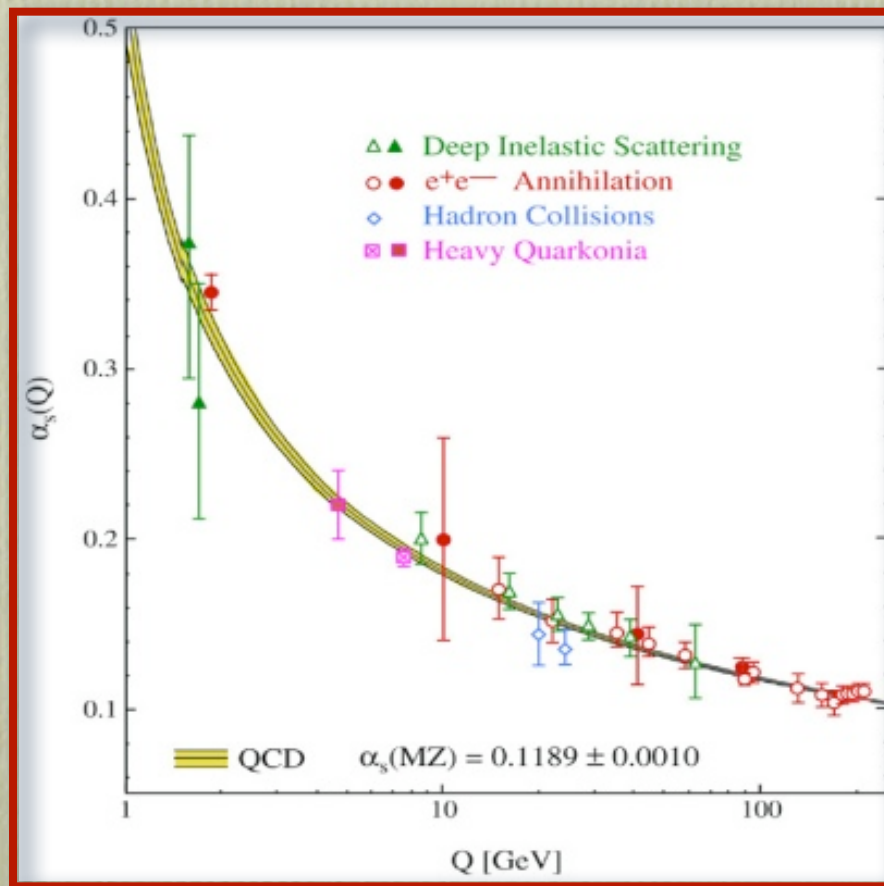
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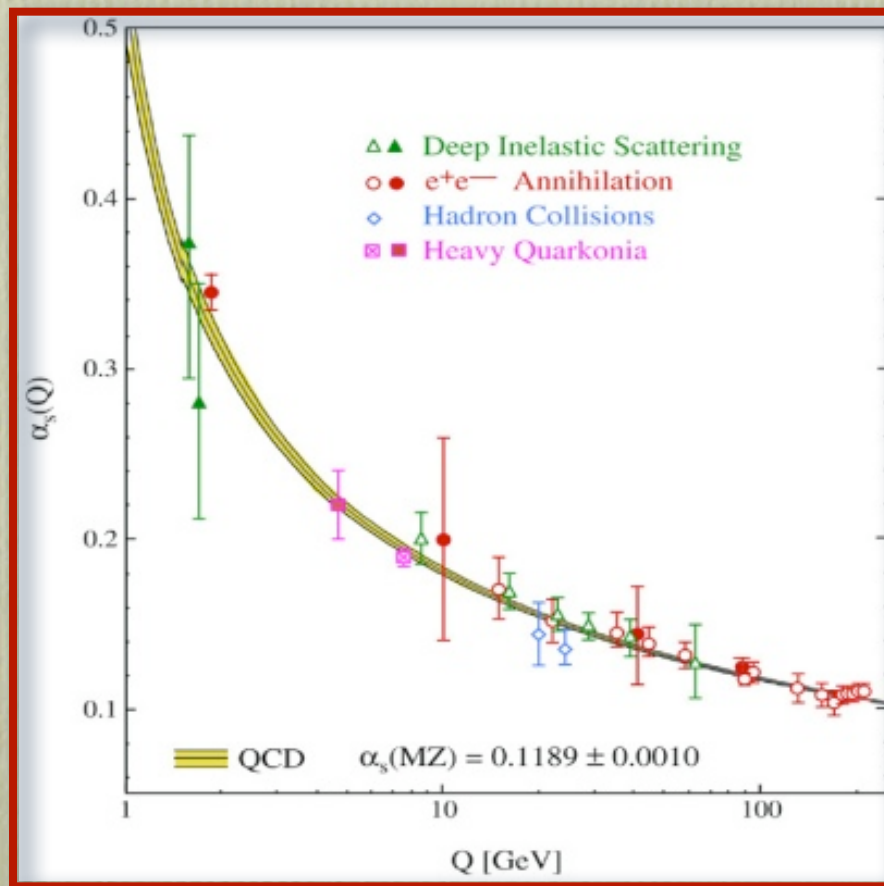
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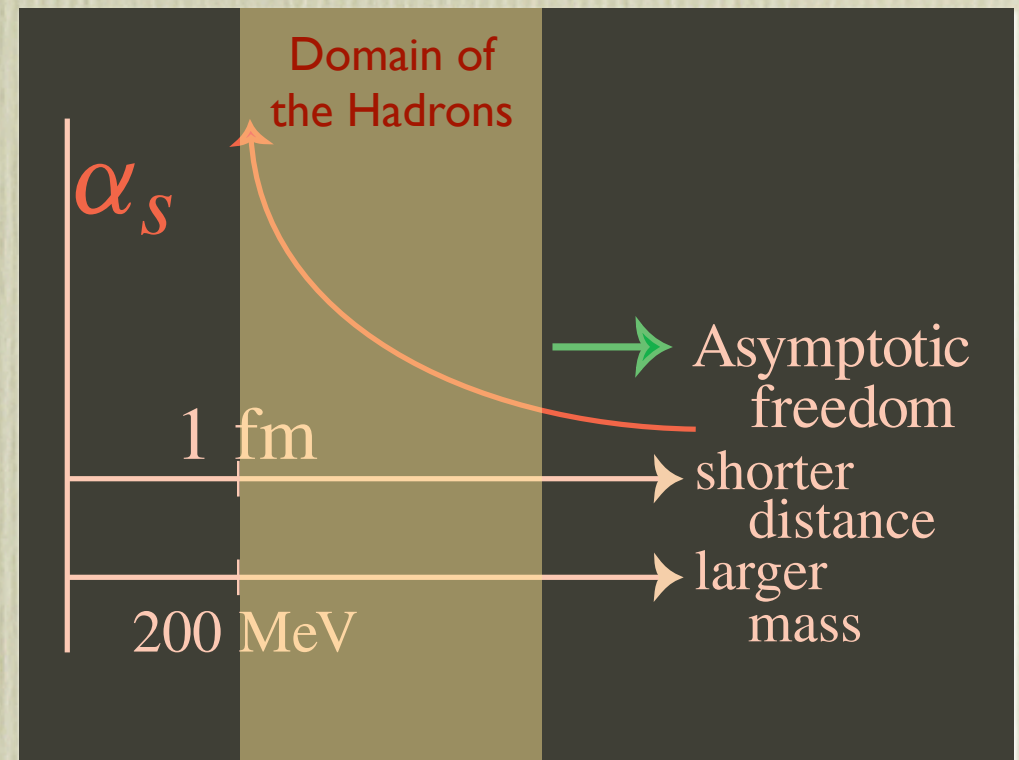
Siegfried Bethke, arXiv:[hep-ex/0606035](https://arxiv.org/abs/hep-ex/0606035)

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- There are light quarks, heavy quarks, and the strange quark

- The strange quark mass is very close to the dynamical mass scale of QCD

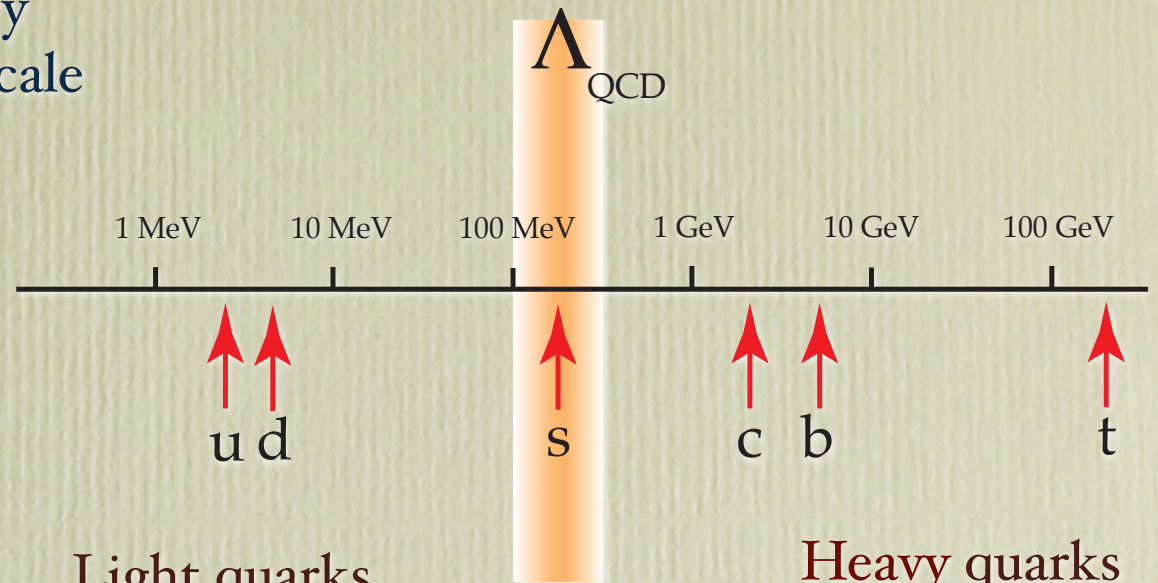
- The electric and weak charge matrices of the quarks are traceless in flavor SU(3)

$$\begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}$$

Light quarks
chiral dynamics

Strange quark

Heavy quarks
non-relativistic
potential theory



- So measurements of electromagnetic and weak charges of hadrons does not reveal their strange quark content. Many mysteries of QCD can be traced to this source. We would know a lot more about strange quarks if their charge was $+2/3$!

Some fundamental questions in QCD...

- IS THERE ARE QUARK MODEL OF HADRONS?
 - QUALITATIVE DESCRIPTION OF HADRONS WHICH LED TO THE DISCOVERY OF QCD HAS BEEN IN TROUBLE FOR SOME TIME, AND HAS RECENTLY SUFFERED ANOTHER BLOW.
- WHAT ARE THE REGULARITIES OF THE HADRON SPECTRUM AND WHAT DO THEY MEAN?
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 - PARITY DOUBLING AND OTHER REGULARITIES OF THE MESON SPECTRUM.
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- (2) Effective field theories. Especially chiral dynamics, the theory of pions interacting with themselves and other hadrons. Systematic if not exact.
- (3) Phenomenological models. Heuristic, historically important. Can they continue to be useful in the era of (1) and (2)?

But, How to use these powerful tools?

But, How to use these powerful tools? Solve QCD!!



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- Change the number of colors.
- Vary the masses of the light quarks.
- Introduce static external color charges.
- Alter the quark color representations and/or electric charges.



Examples

1. A new lattice calculation of the orbital angular momentum on the up and down quarks in the proton will shock the spin physics community.

Ph. Hagler et al. (LHPC Collaboration), arXiv:0703.4295

2. Varying the number of colors in QCD and the masses of the u and d quarks shows that the σ meson – so important in the nucleon-nucleon force and the binding of nuclei – is not like other mesons. It is a multiquark state.

J. R. Pelaez, arXiv:hep-ph/0307018, arXiv hep-ph/0309292, arXiv hep-ph/0510118; R. Jaffe, arXiv hep-ph/0701038; C. Hanhart, J. R. Pelaez, and G. Rios, arXiv 0712.0473

3. Introducing external static color sources reveals diquark and other correlations in QCD.

R. Jaffe, hep-ph/0507149, C. Alexandrou, Ph. de Forcrand, and B. Lucini, arXiv hep-lat/0609004

4. Varying quark masses explores the *congeniality* of other Universes in the multiverse!

I. Kimchi, A. Jenkins, and R. L. Jaffe, in preparation

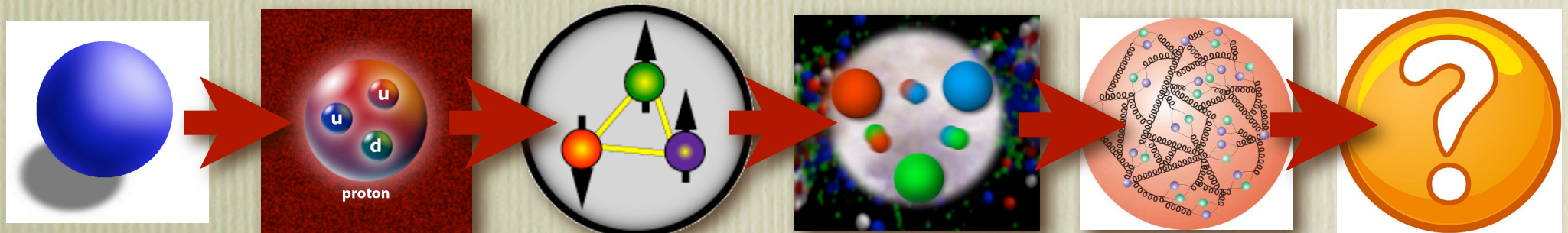
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- 1970's --- incorporation into QCD, refinement
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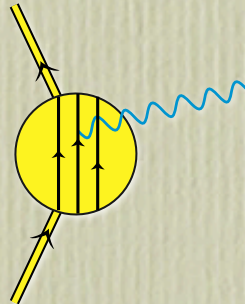
1. The Proton is Made of Quarks

- ★ Most naive model: Three non-relativistic quarks
- ★ Simplest quark model wavefunctions

$$|p^\uparrow\rangle = \frac{1}{\sqrt{6}} (2|u^\uparrow u^\uparrow d^\downarrow\rangle - |u^\uparrow u^\downarrow d^\uparrow\rangle - |u^\downarrow u^\uparrow d^\uparrow\rangle)$$

$$|n^\uparrow\rangle = \frac{1}{\sqrt{6}} (2|d^\uparrow d^\uparrow u^\downarrow\rangle - |d^\uparrow d^\downarrow u^\uparrow\rangle - |d^\downarrow d^\uparrow u^\uparrow\rangle)$$

- ★ Classic prediction of magnetic moment ratio



$$\frac{\mu_n}{\mu_p} \cong -\frac{2}{3} \cong -\frac{1.91}{2.79}$$

- ★ Successes like μ_n/μ_p led to more detailed quark model building, to the development of **QCD** and eventually, to predictions of the proton and neutron spin content.

2. The First “Spin Crisis”

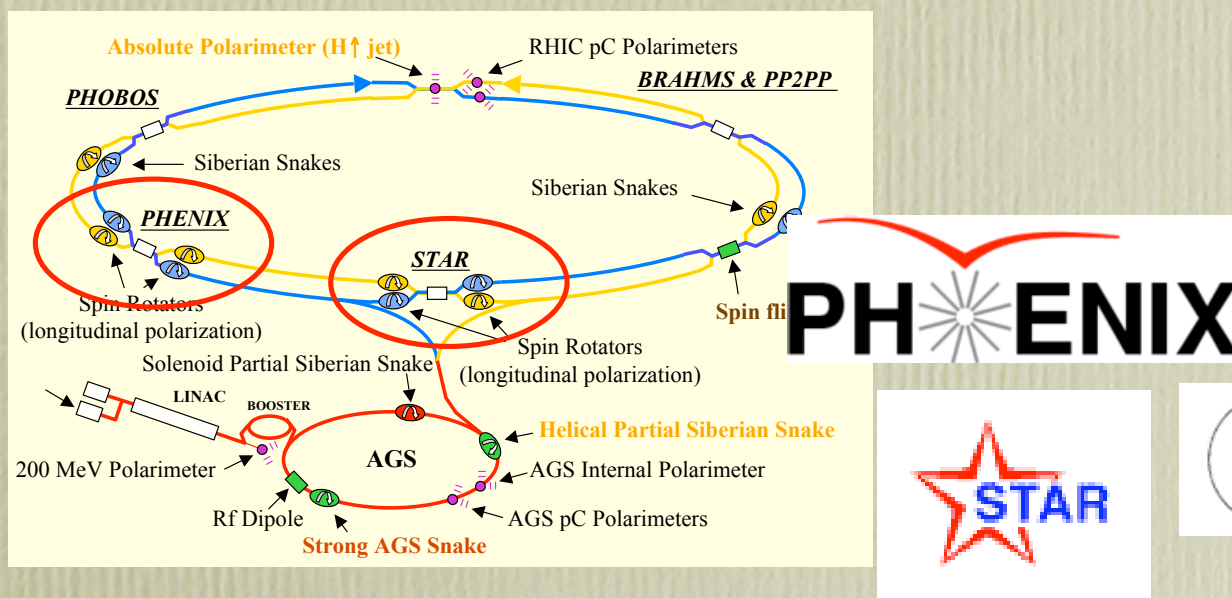
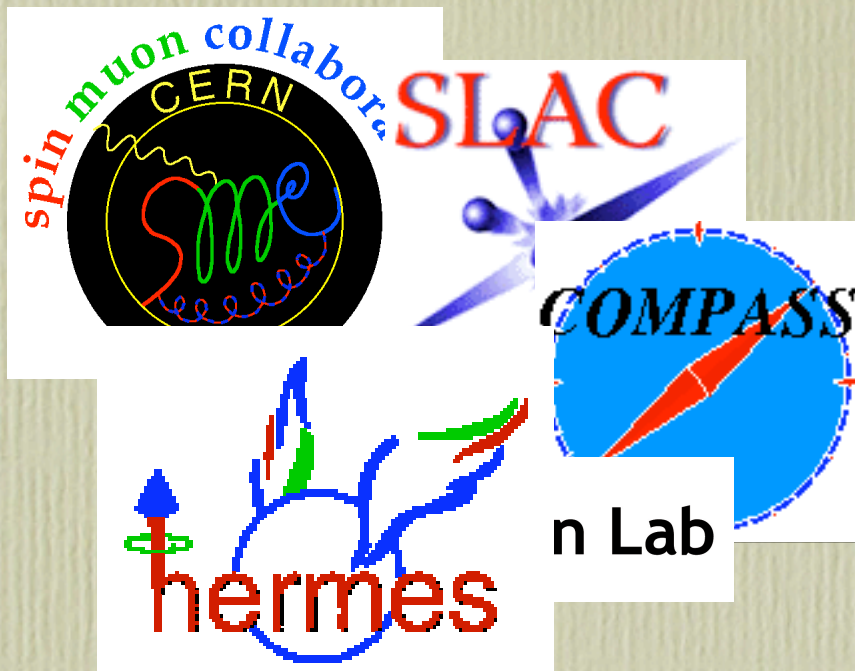
- ★ On general grounds the angular momentum of the proton is the sum of contributions of spin and orbital angular momentum of both quarks and gluons

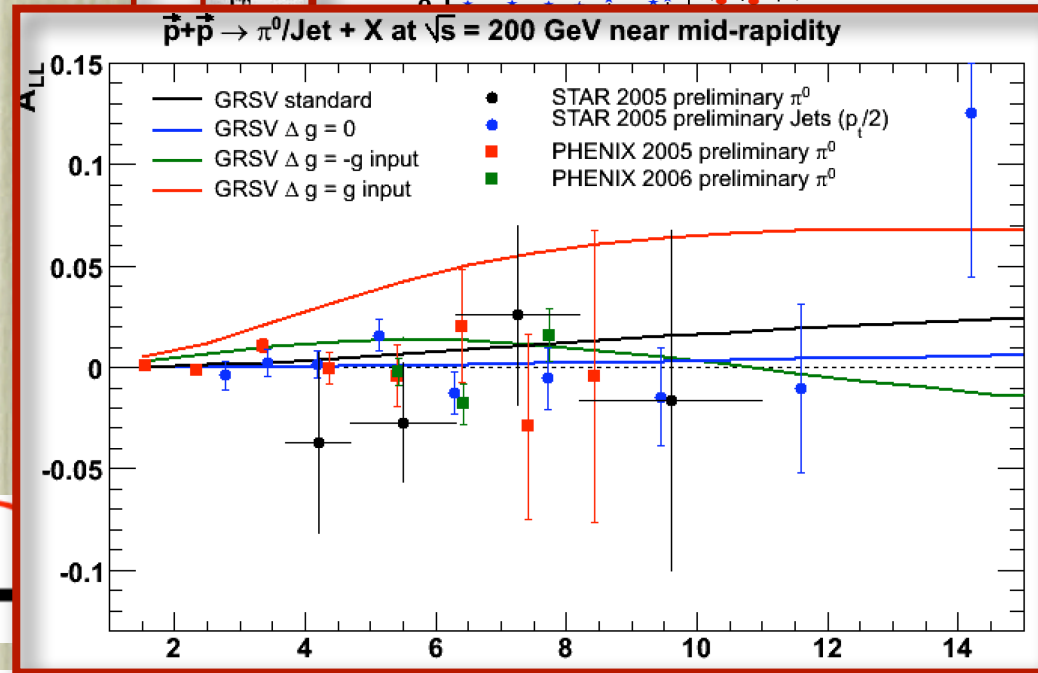
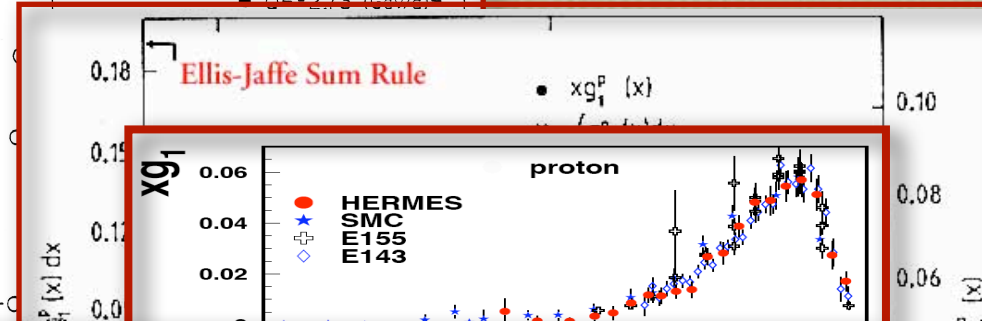
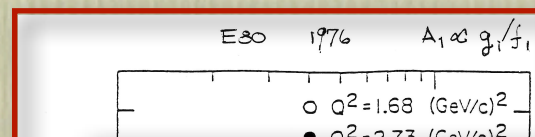
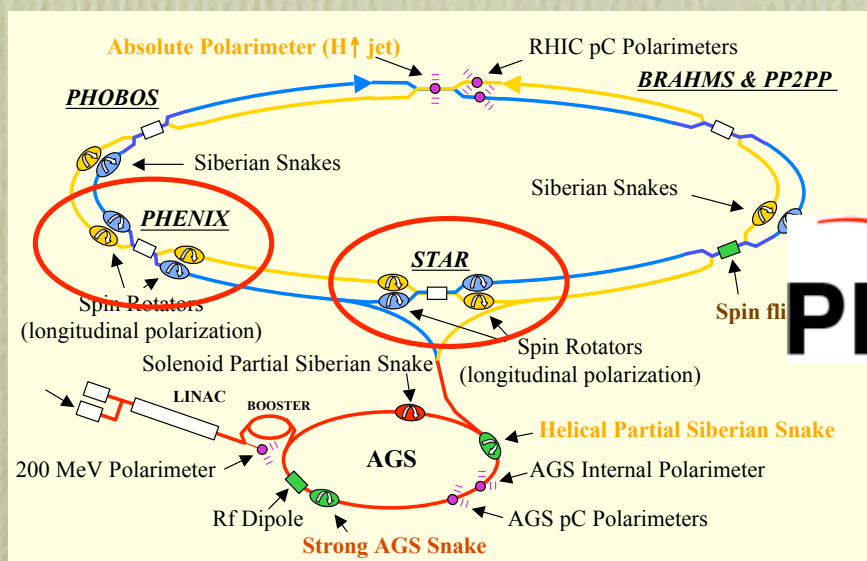
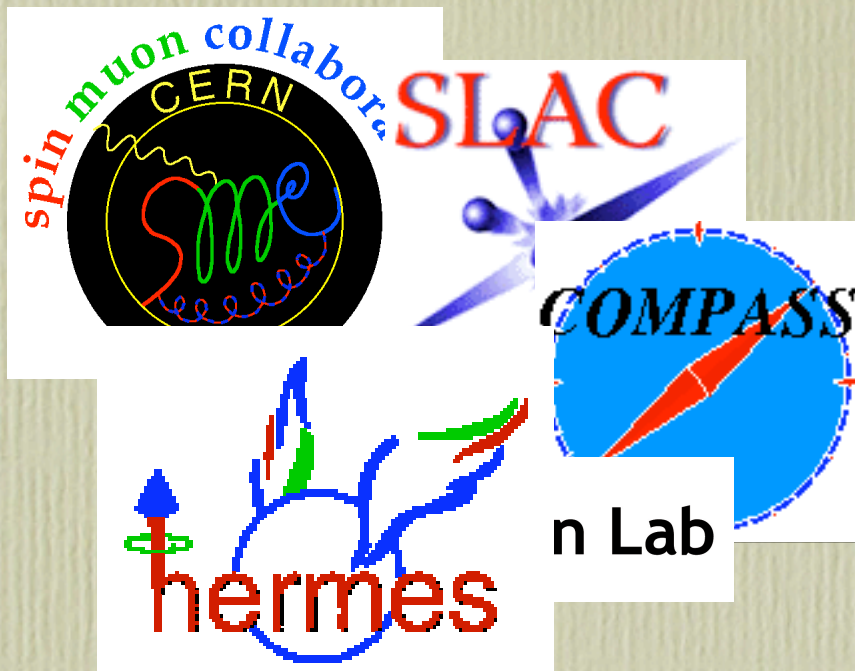
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \Delta L_q + \Delta L_g$$

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

- ★ The changing picture of the proton spin...







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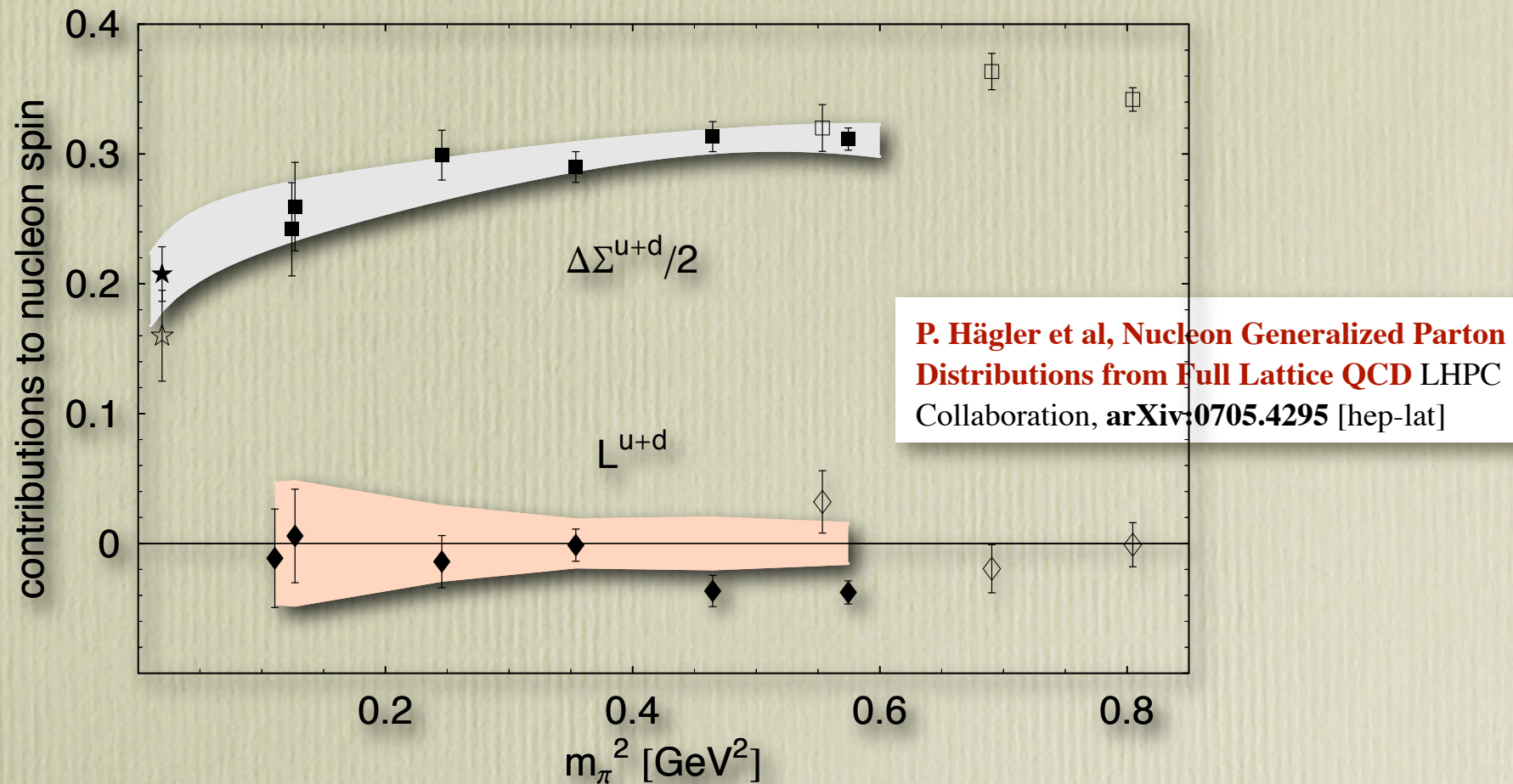
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- ★ This one engendered by the first **lattice QCD calculation** of $\langle L_q \rangle$ for ***u*** and ***d*** quarks.
- ★ First (mild) surprise: Quark orbital angular momentum is consistent with zero.

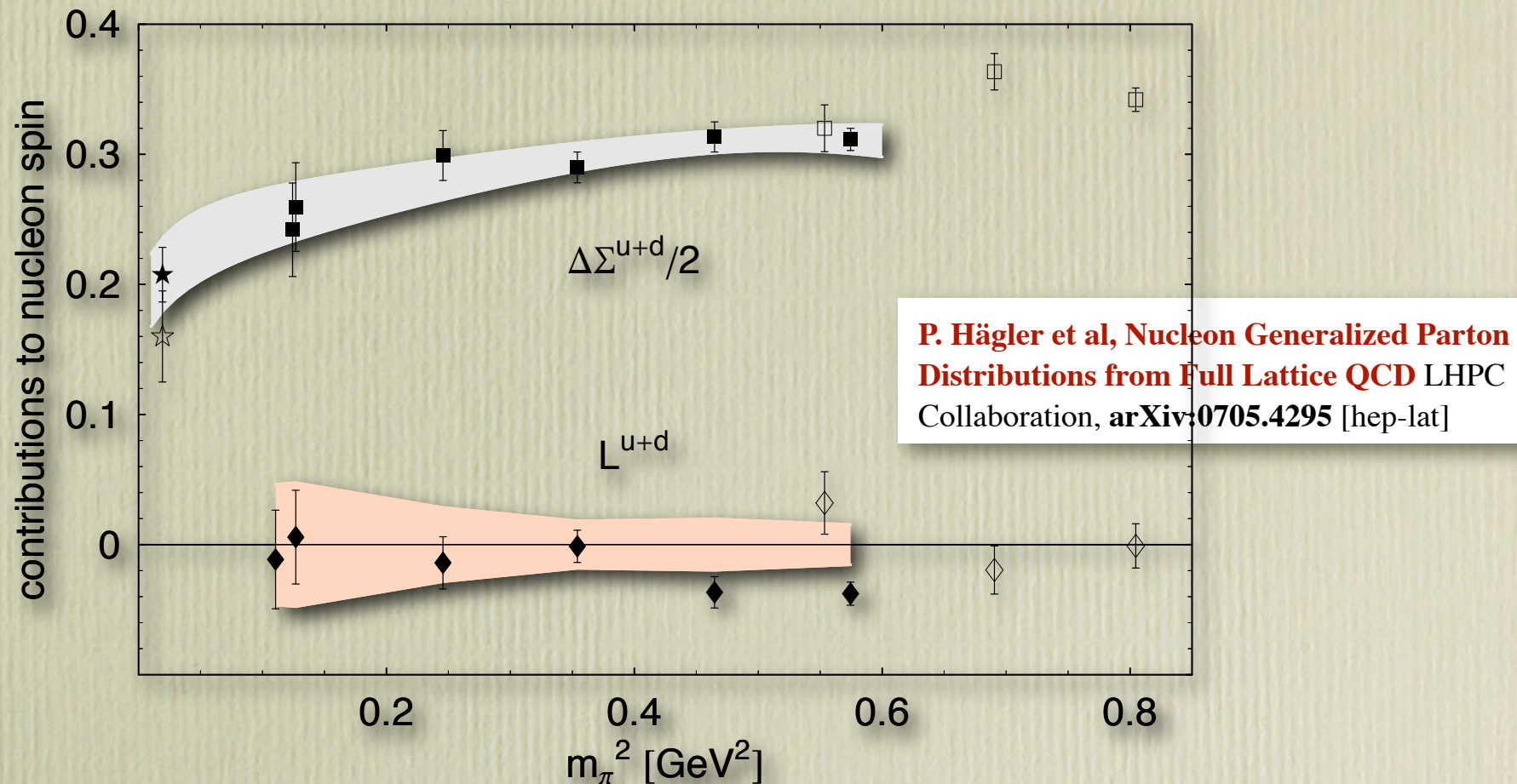
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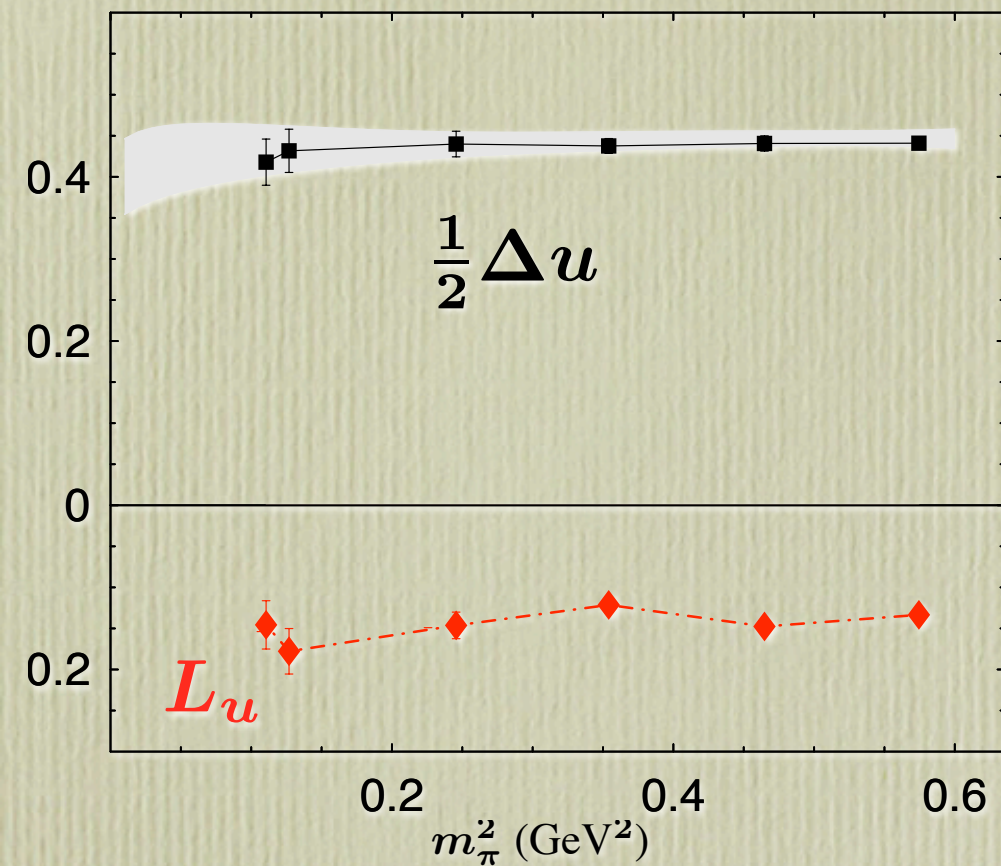
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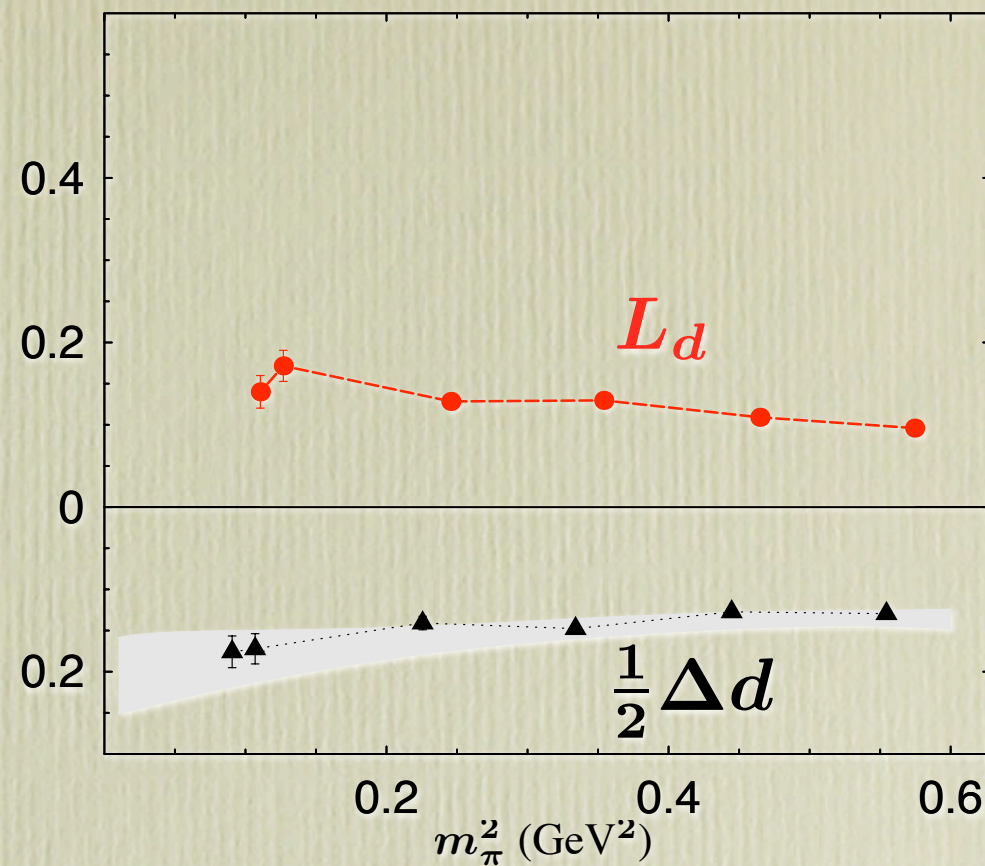
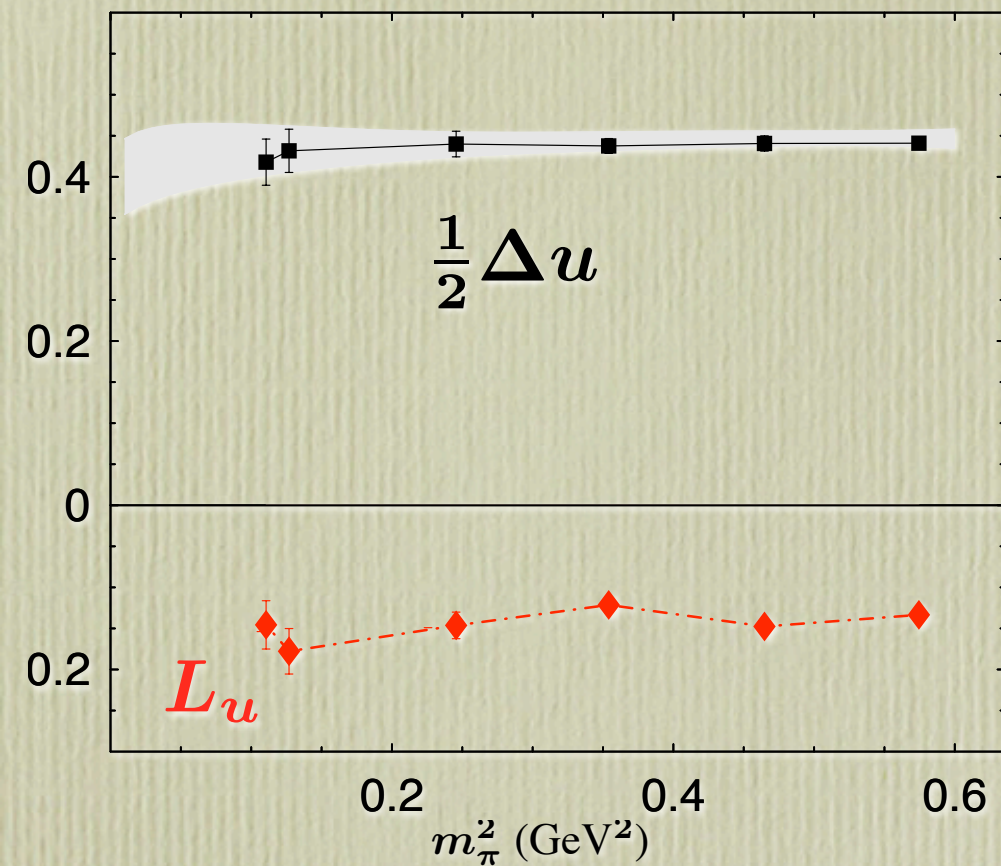
Disconnected diagrams are omitted, so contributions of $q\bar{q}$ pairs to L_q are possible.

Hägler et al also calculate L_u and L_d separately.

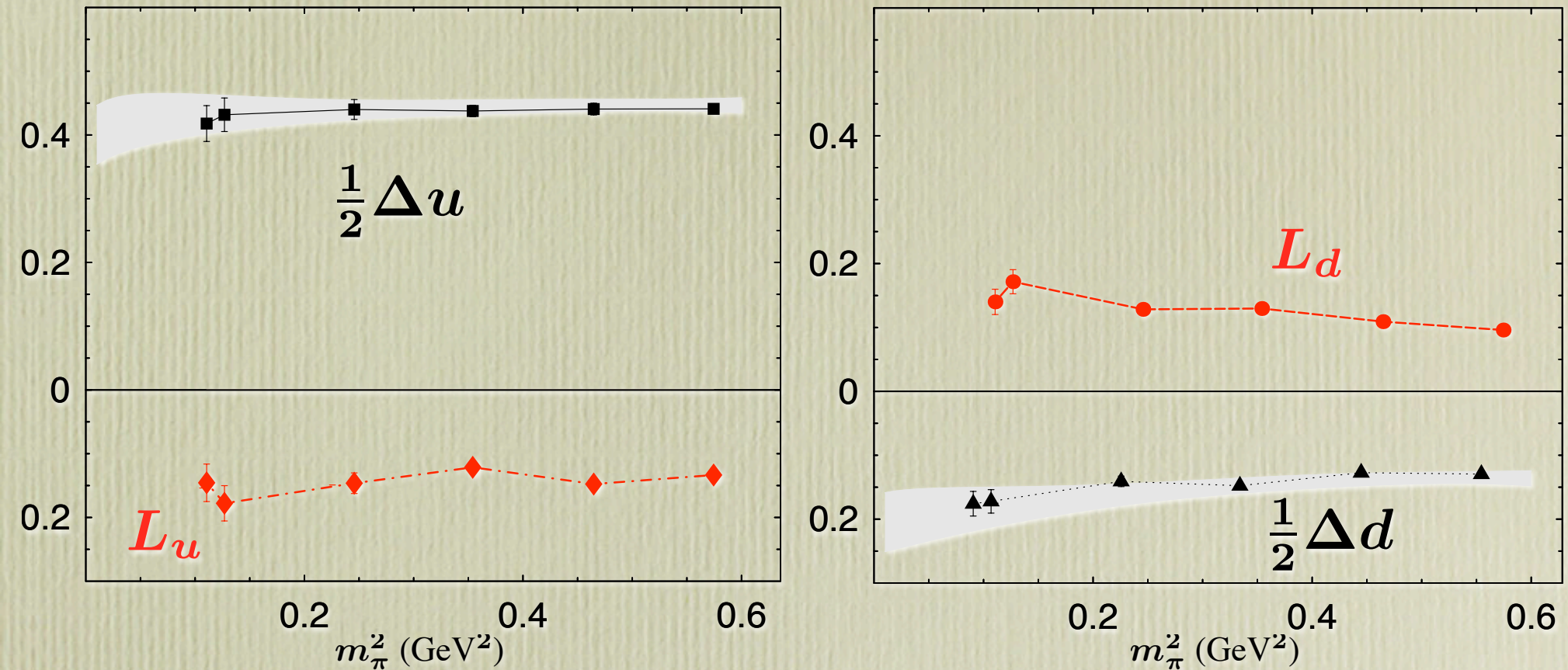
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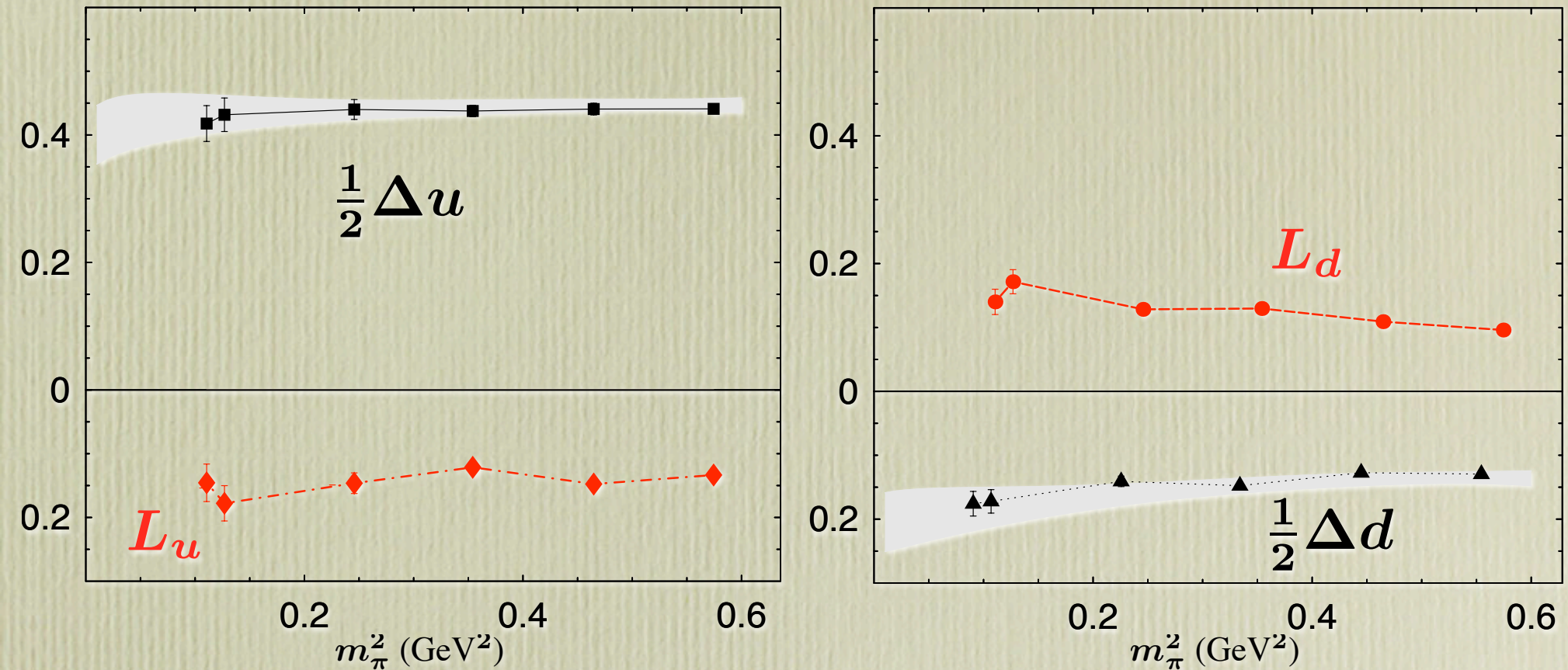
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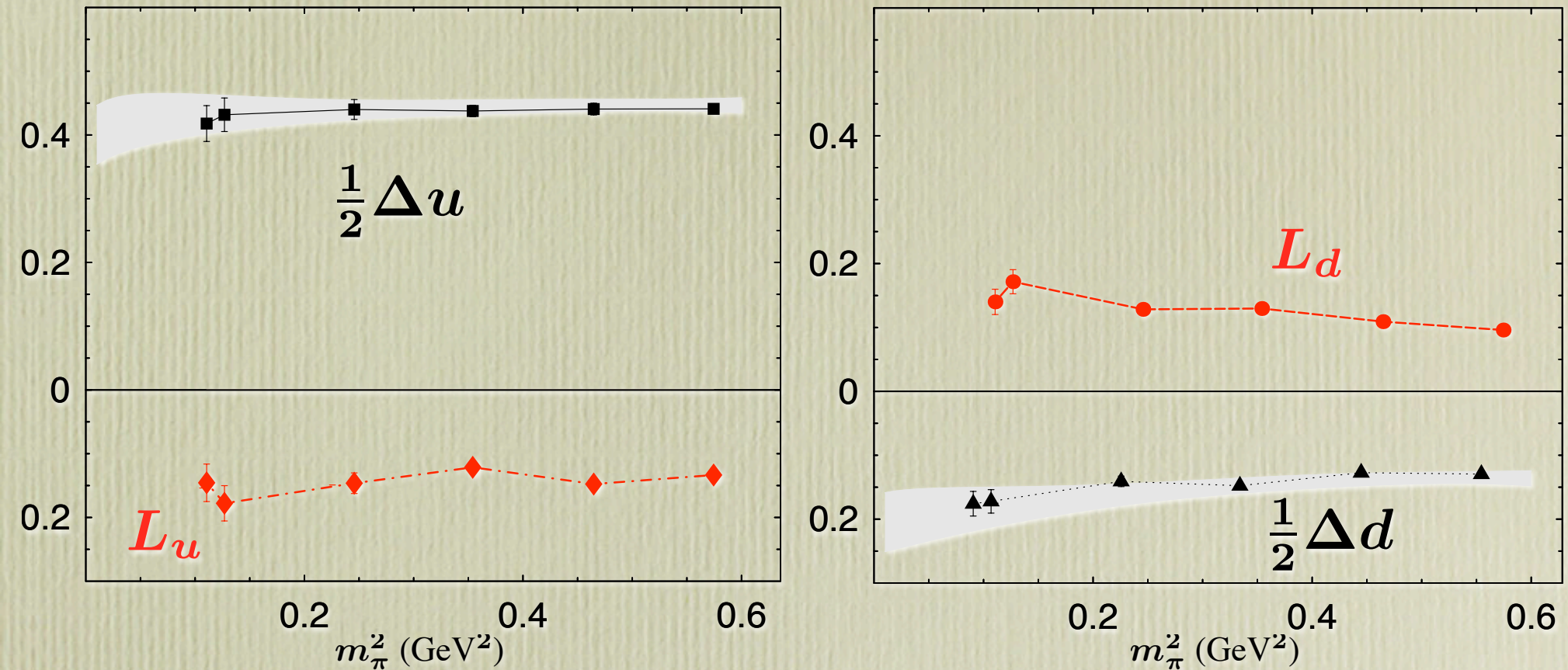
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This is the opposite of what is seen!

Note that $L_u - L_d$ is an isovector and not effected by disconnected diagrams

Everything's coming out zero!

$$\Delta u + \Delta d + \Delta s \approx 0$$

$$\frac{1}{2}\Delta u + L_u \equiv J_u \approx 0$$

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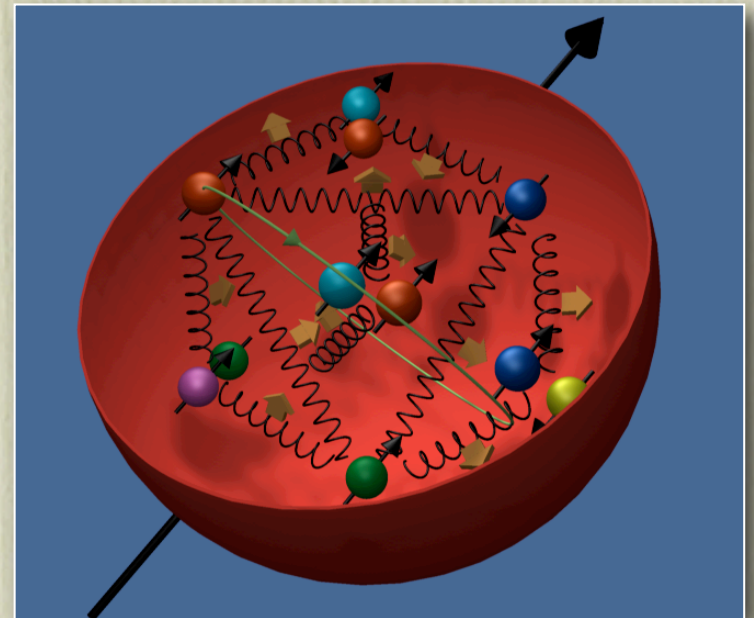
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So, what remains of the quark model?
And, where is the proton's spin?



II. Varying N_c reveals two types of hadrons

Ancient question: How should enhancements and resonances be described in hadronic scattering processes?

| | <u>Ordinary Hadrons</u> | <u>Extraordinary Hadrons</u> |
|---|-------------------------|------------------------------|
| • Are they associated with poles in the S-matrix? | • YES | • NOT NECESSARILY |
| • Can they be parameterized by some simpler formalism, eg. as poles in the “K-matrix”? | • YES | • NO |
| • Are they generated by “forces” between the scattering hadrons? | • NO | • LIKELY YES |
| • Is there some underlying “zero width” approximation that is a good approximation to the physical situation? | • YES | • NO |
| • Does one size fit all? | <u>NO!</u> | |

Qualitative answers:

R. L. Jaffe, arXiv hep-ph/0701038

- “**Ordinary hadrons**” are **Feshbach Resonances** — bound states in confined channels that appear in the continuum of scattering channels. **Their widths vanish as $N_c \rightarrow \infty$** . Almost any zero width approximation (eg. the K matrix) is a good starting point. The ρ meson is a classic example.
- But there are other hadrons of great interest: “**Extraordinary hadrons**” that resemble **unitarized potential resonance** of the old bootstrap. **They disappear as $N_c \rightarrow \infty$** . No zero width approximation applies. Examples include the famous σ (or $f_0(600)$) of $\pi\pi$ scattering and some of the new charmonium states seen at B factories.

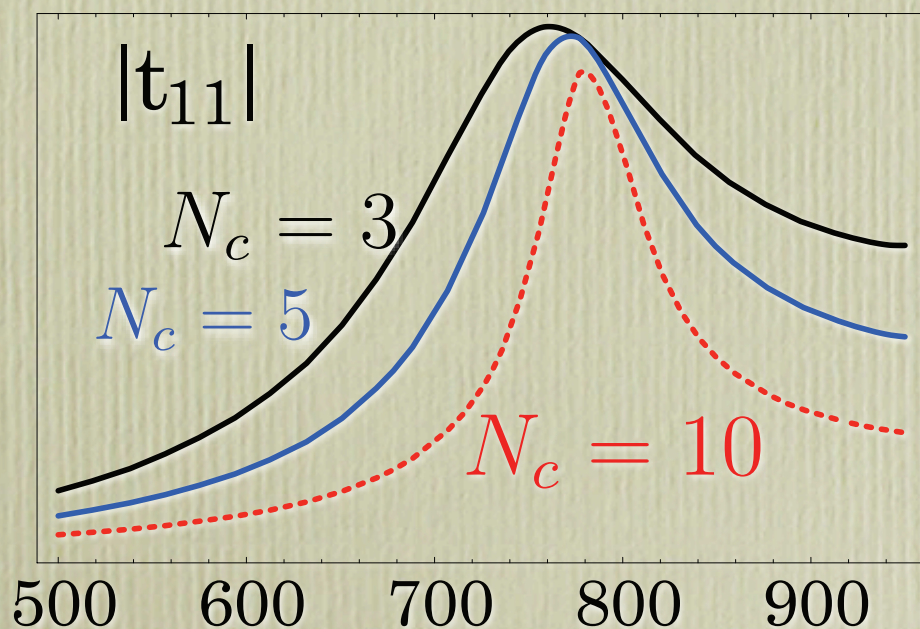
N_c and quark mass dependence of low energy $\pi\pi$ scattering from chiral dynamics

J. R. Pelaez, arXiv:hep-ph/0307018, hep-ph/0309292, hep-ph/0510118
C. Hanhart, J. R. Pelaez, and G. Rios, arXiv 0712.0473

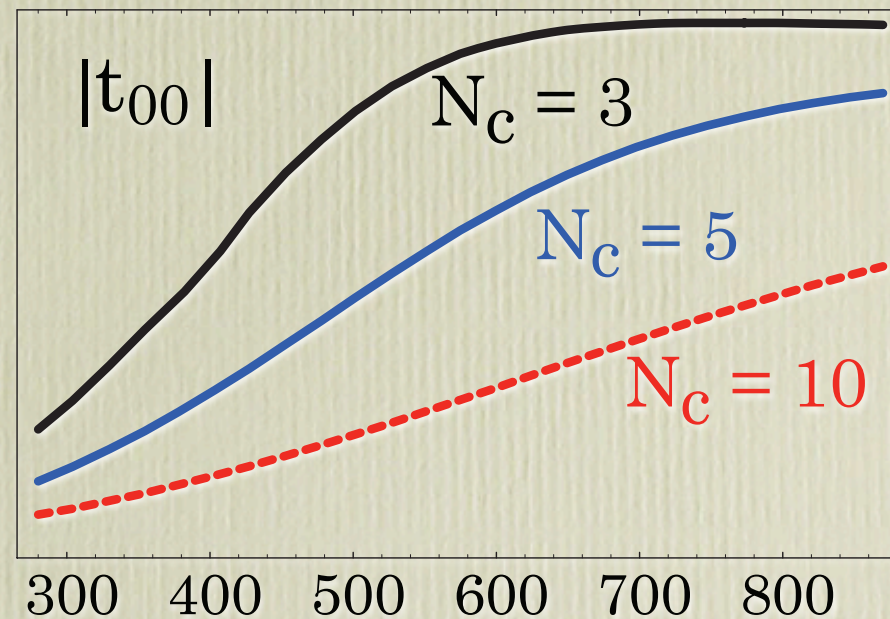
Chiral dynamics is sufficiently accurate to study the dependence of the ρ -meson and σ -meson mass and width on number of colors and on quark masses.

Although $N_c = 3$ in our world, N_c dependence reveals striking difference between these two enhancements.

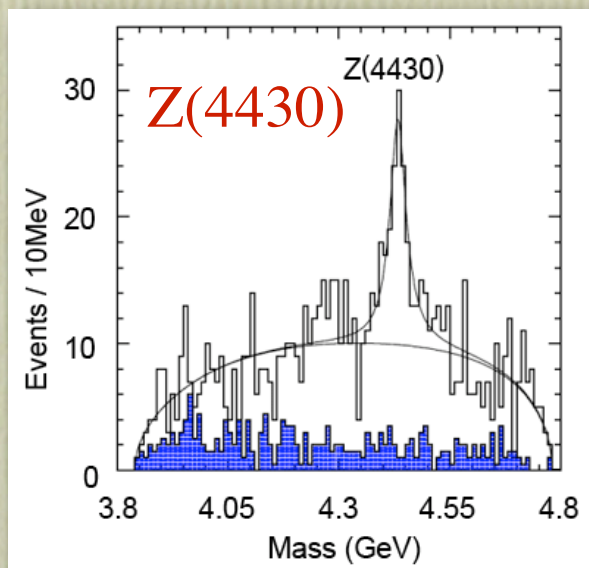
ρ Region



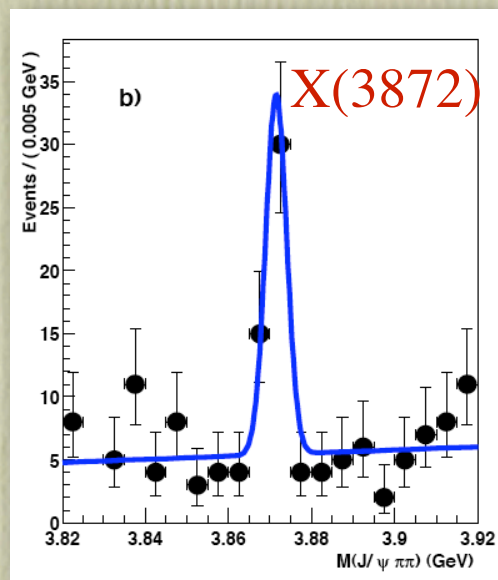
σ Region



Unexpected charmonium states seen in B-decays

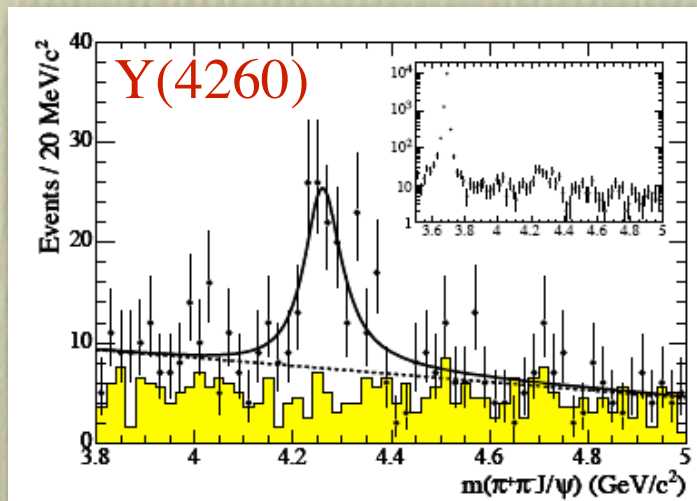


Belle

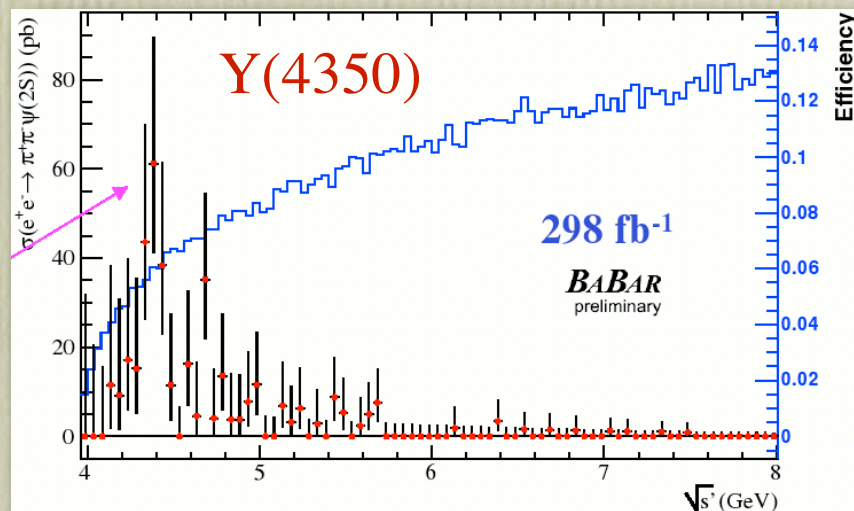


Belle

The new, unexpected charmonium states seen at Belle and Babar have many properties characteristic of extraordinary hadrons. The Z(4430) is particularly interesting because it decays to $J/\Psi \pi^+$ which is characteristic of a multiquark state.



BaBar



BaBar

III. Introducing external, fixed color sources into QCD allows the study of quark correlations.

R. Jaffe, hep-ph/0507149

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Color non-singlet spectroscopy on the lattice

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Baryons

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Pentaquark
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Color non-singlet quark source and sink

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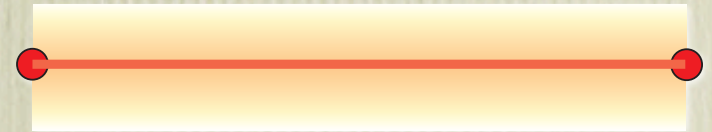
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Static, infinitely massive,
neutralizing antitriplet =
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Color non-singlet spectroscopy on the lattice

C. Alexandrou, Ph. de Forcrand, and B. Lucini,
arXiv hep-lat/0609004

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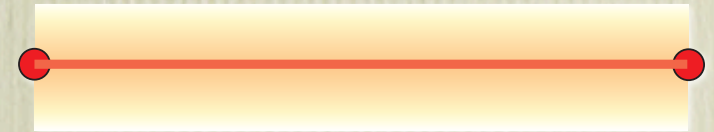
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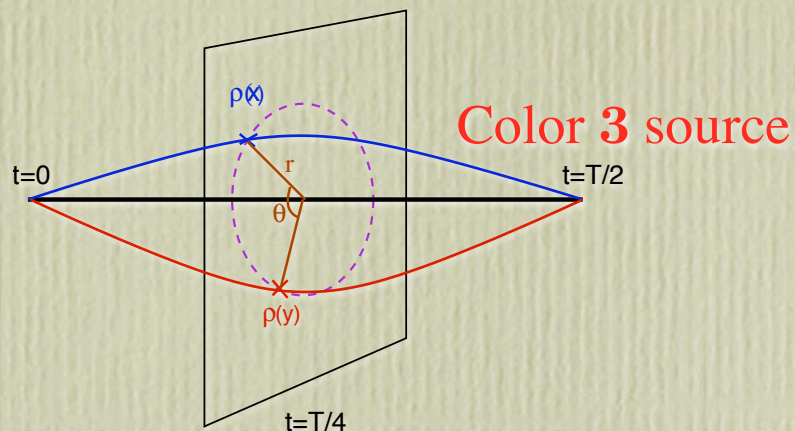
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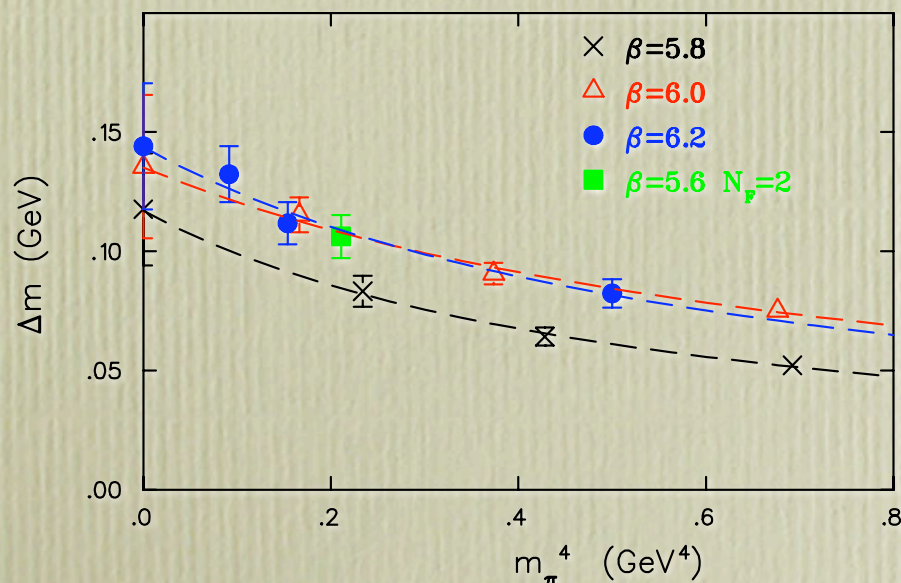


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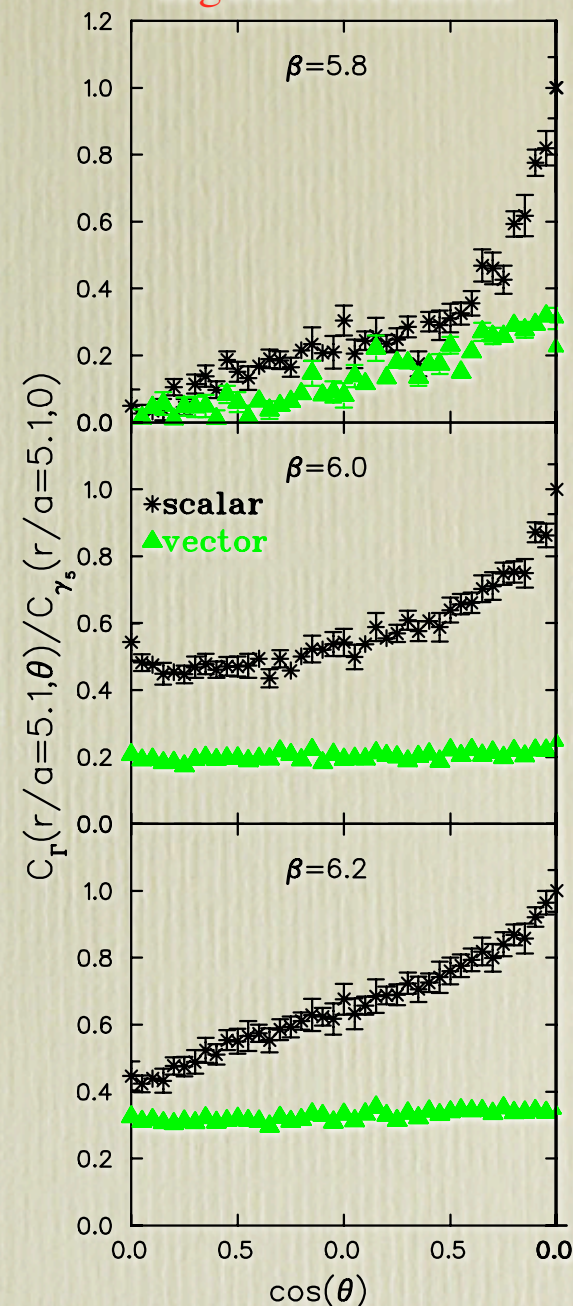
Alexandrou and de Forcrand look at mass difference and spatial correlation of “Good” and “bad” diquarks in the background of a fixed color triplet source



Good-bad diquark mass difference as a function of m_π^4



Good-bad diquark angular correlation



IV. Quark mass deformation of QCD.

- Quark mass variation is one of the deformations that can expose underlying dynamics in QCD. Topics include: Spin splittings, nuclear forces, light quark resonance interactions.
- And relates to an entirely different issue:
 - How special are the interactions and parameters that characterize our Universe?
 - Is it possible that our physical laws are environmentally selected?
 - How does QCD phenomenology change as the number, electric charges and especially, the masses, of light quarks change?
- Much discussed recently.* Our understanding of QCD allows us to make a definitive answer for very interesting special cases.
- Search for domains of “congenitality” (= possibility of evolving an observer) in the parameter space of the Standard model.

* S. Weinberg, **PRL 59, 2607 (1987)**, hep-th/0511037; V. Agrawal, S. M. Barr, J. F. Donoghue, D. Seckel, **PRD 57, 5480 (1998)**, **PRL, 80, 1822 (1998)**; C. J. Hogan, **astro-ph/06022104**; L. Hall and Y. Nomura, **0712.2454**.

The parameter space of the standard model is very complicated.

We consider the congeniality of worlds on a slice as we

Vary light quark masses with as much as possible held fixed.

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An exercise to see how hard this kind of physics will be:

- Vary the light (u, d, s) quark masses over domain where $SU(3)_f$ perturbation theory is valid — in practice $m_u + m_d + m_s \lesssim 150$ MeV.
- Redefine the MeV so the average octet baryon mass stays equal to its value in our world.
- Adjust the electron Yukawa coupling so the electron mass stays fixed.
- Adjust Λ_{QCD} to keep the strength of the nuclear binding energy fixed.

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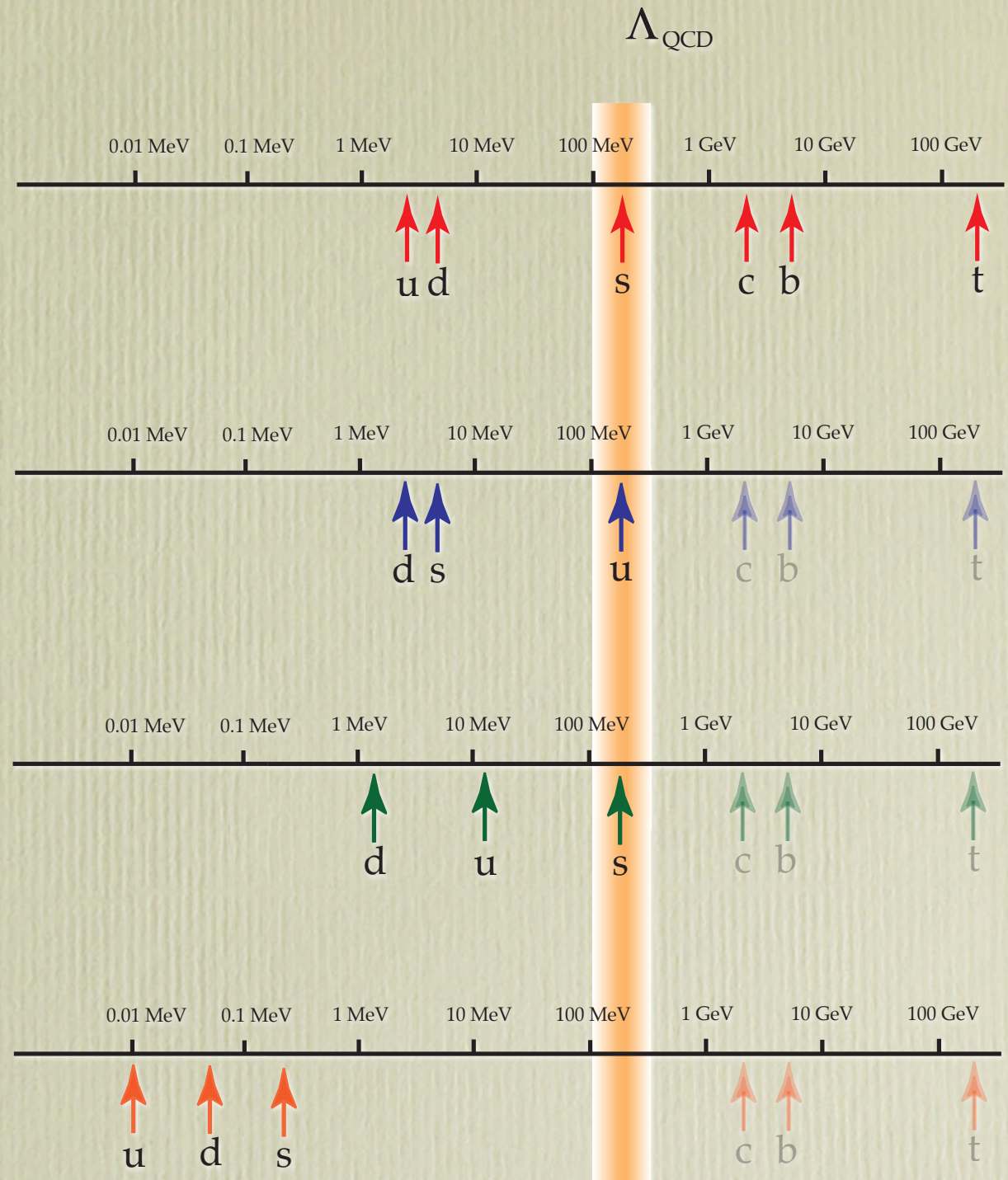


OUR WORLD

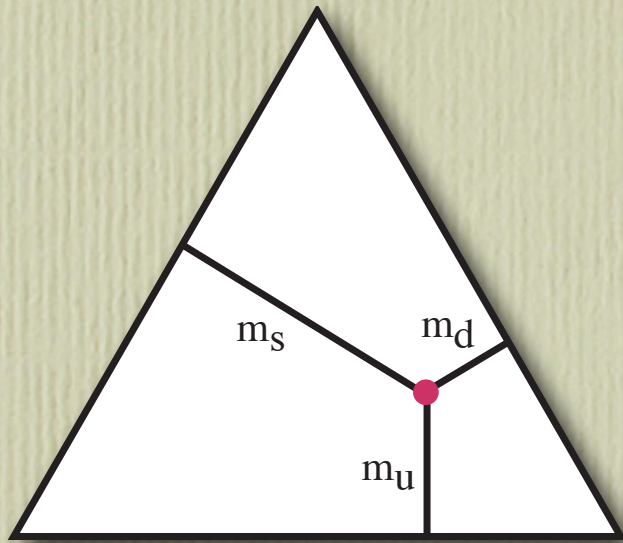
A WORLD WHERE THE LIGHT
QUARKS THAT FORM NUCLEI
BOTH HAVE CHARGE $-1/3$

A WORLD WHERE THE UP
QUARK IS CONSIDERABLY
HEAVIER THAN THE DOWN
QUARK

A WORLD WHERE ALL THREE
LIGHT QUARKS ARE SO LIGHT
THAT ALL EIGHT BARYONS
PARTICIPATE IN NUCLEAR
PHYSICS (THE DREAM WORLD
OF HYPERNUCLEAR PHYSICS)

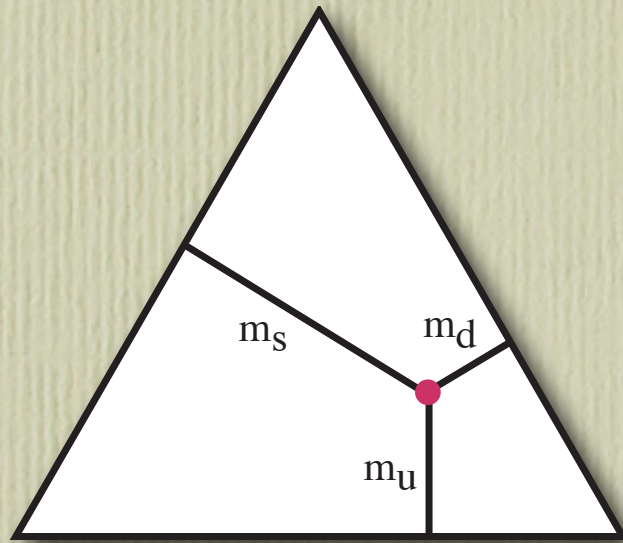


Parameterizing the space of possibly light quark worlds



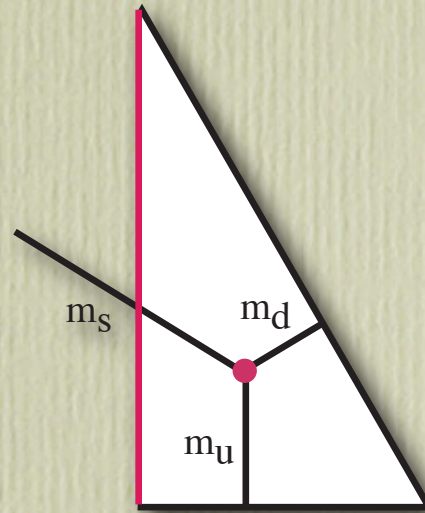
$$m_u + m_d + m_s = m_0$$

Parameterizing the space of possibly light quark worlds

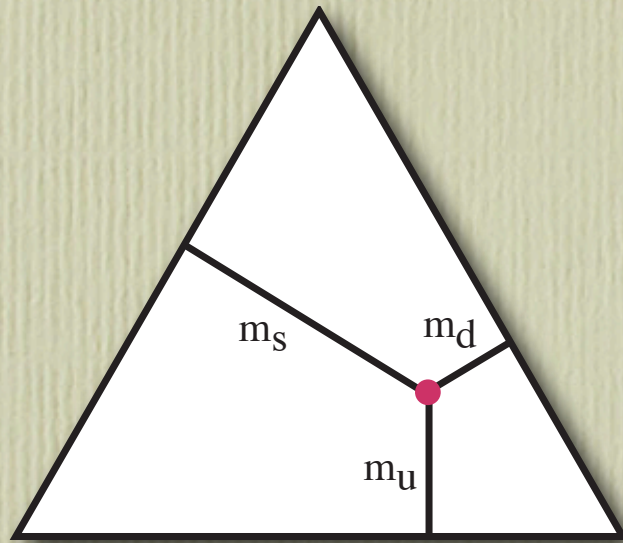


$$m_u + m_d + m_s = m_0$$

$m_s \Leftrightarrow m_d$
symmetry

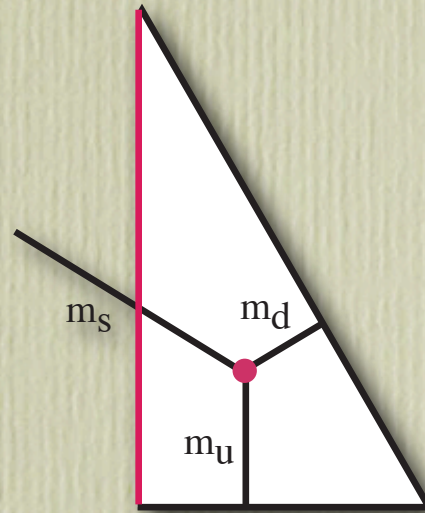


Parameterizing the space of possibly light quark worlds

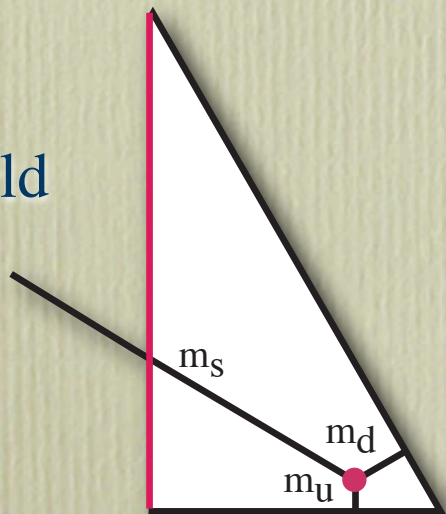


$$m_u + m_d + m_s = m_0$$

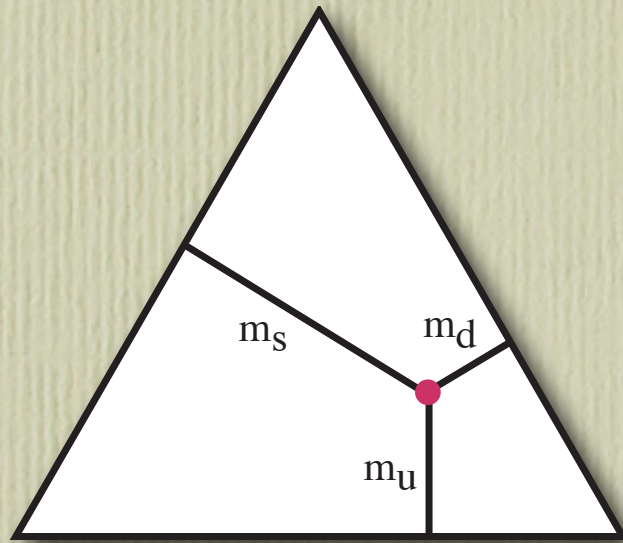
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Our world

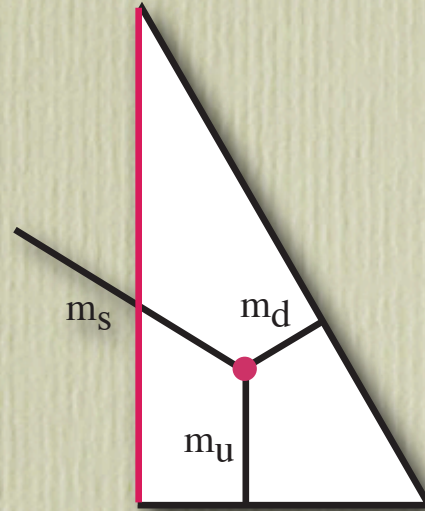


Parameterizing the space of possibly light quark worlds

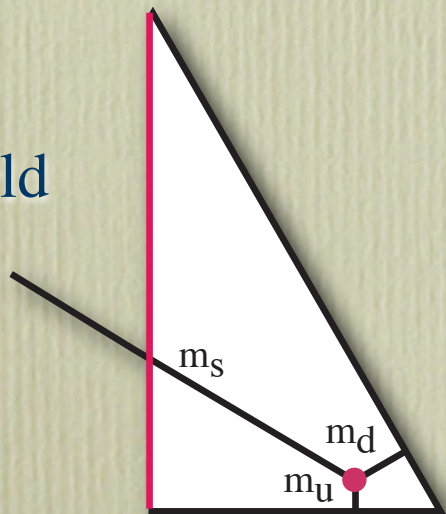


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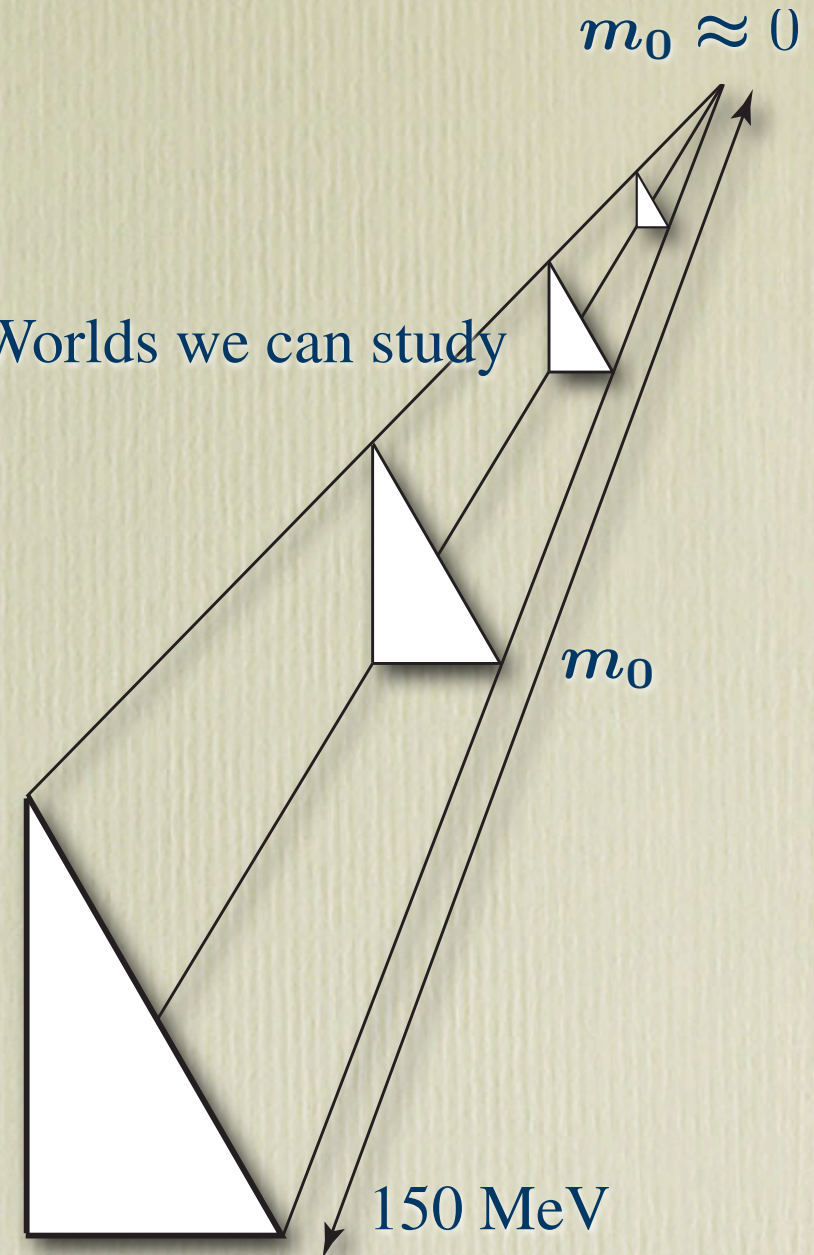
$m_s \Leftrightarrow m_d$
symmetry



Our world



Worlds we can study



Some qualitative results...

